LESSON PLAN

INTELLIGENT INTELLIGENCE

High School (9-12)

OVERVIEW:

The United States military and intelligence community relies on a variety of techniques to collect information. While you may not get this impression from spy movies, many of those techniques use math. In this lesson, students will be introduced to formulas used in imaging and radio communication and apply them to intelligence-gathering scenarios.

Note: The formulas used throughout the lesson are relatively simple; however, some problems use numbers that are incredibly large or incredibly small. Calculators and computers will vary on their level of precision when calculating these values, so when checking student answers there may be some variation in the tenths and hundredths.

Estimated time: 60 minutes

STANDARDS:

Common Core Standards

- CCSS.MATH.CONTENT.HSA.SSE.A.1: Interpret expressions that represent a quantity in terms of its context.
- CCSS.MATH.CONTENT.HSA.CED.A.1: Create equations and inequalities in one variable and use them to solve problems.
- CCSS.MATH.CONTENT.HSA.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- CCSS.MATH.CONTENT.HSA.REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- CCSS.MATH.CONTENT.HSA.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

LEARNING OBJECTIVES:

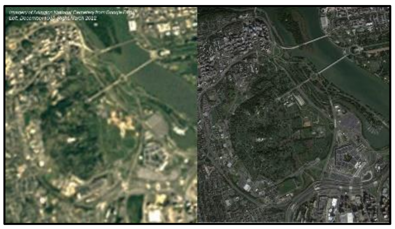
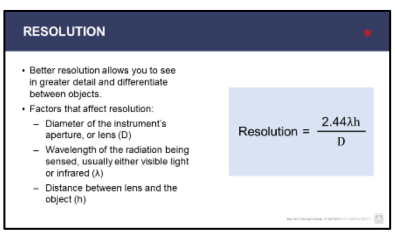
- Students will complete algebraic problems using provided formulas.

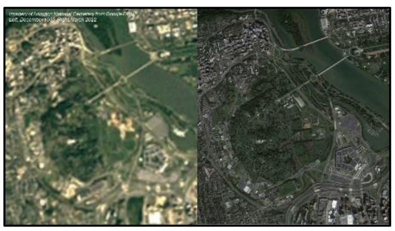
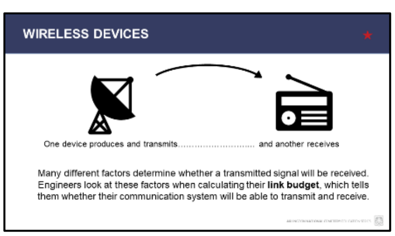
RESOURCES NEEDED:

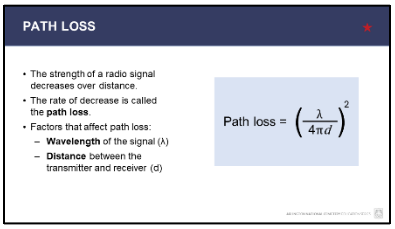
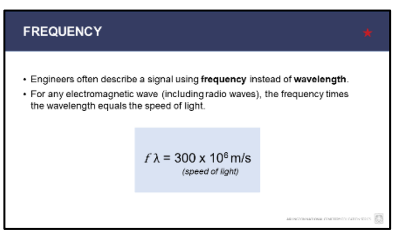
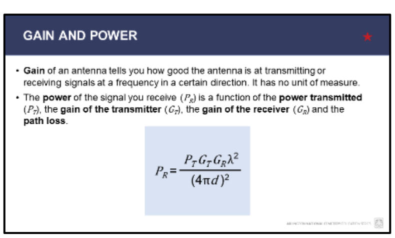
1. Intelligent Intelligence PowerPoint
2. Intelligent Intelligence: Satellite Imagery worksheet (1 per student)
3. Intelligent Intelligence: Radio Communications worksheet (1 per student)

LESSON ACTIVITIES:

- Use the notes below to introduce the formulas necessary to complete the student worksheets. Guide students through each problem as necessary.

Slide:	Notes for Presentation
Slide 2: Satellite Images 	Gathering intelligence requires a lot of math. In this lesson, we will practice using some of the formulas used in imaging and radio communication and apply them to intelligence-gathering scenarios. Reconnaissance satellites, or spy satellites, are valuable tools because they can collect and transmit all sorts of information, 24/7, from the safety of orbit. One type of information they can gather is imagery. Compare these two satellite images. Which one shows more detail?
Slide 3: Resolution  	Another way to describe that difference in level of detail, or sharpness, is resolution. Better resolution allows you to see more detail and differentiate between objects. If a camera's resolution is .01 meters, that means it can differentiate between objects that are .01 meters apart, or detect objects that are .01 meters in size. When comparing resolutions, a smaller number reflects better resolution. To take a concrete example: When you look at a tree that is far away you can't see the individual leaves, but you would be able to see them if you used binoculars. The binoculars allow you to see in better resolution and therefore differentiate between the individual leaves. There are a few different factors that affect resolution: 1. The diameter (D) of the instrument's aperture, or lens 2. The wavelength (λ) of the radiation being sensed, in most cases either visible light or infrared

	3. The distance (h) between the lens and the object Going back to the tree example, the binoculars have a better resolution than your eyes because the lens on a pair of binoculars is much bigger than the lens in your eye (your iris). If you walked closer to the tree, decreasing the distance between your eye and the leaves, this would also increase your resolution and allow you to differentiate between them. To find the resolution of a camera (or your eye, or any detecting instrument), use this equation: $$\text{Resolution} = \frac{2.44\lambda h}{D}$$
Slide 4: Satellite Images 	Looking at these satellite images again, which one do you think is more valuable for reconnaissance? Camera resolution has a huge impact on the quality of data we can collect from reconnaissance satellites. Let's apply what you now know about resolution to some spy situations. *Hand out the Intelligent Intelligence: Satellite Imagery worksheet. You may choose to allow students to complete the worksheet individually, in groups, or together as a class.* Answers: 1. You would need a resolution of at least 3 meters. 2. Satellite 4 3. 1.82 meters or 182 centimeters 4. 3.42 meters or 342 centimeters 5. 2,013 meters or 2.013 kilometers
Slide 5: Wireless Devices  	*Hand out the Intelligent Intelligence: Radio Communications worksheet. This worksheet will need to be completed in chunks, with explanations of each formula used.* Wireless devices – cell phones, radios, Bluetooth devices, anything using WiFi – communicate via radio waves. One device produces and transmits the radio waves at a given frequency, while another receives them. A wireless device cannot just receive any and all signals, though. There are many different factors that determine whether a transmitted signal will be received, including the power of the transmitted signal, the distance it has to travel, whether the receiver's antenna is sufficient to receive the signal,

	what type of medium the signal has to travel through, etc. Engineers look at all of these factors when calculating their link budget, which tells them whether their communication system will be able to transmit and receive.
Slide 6: Path Loss  	The strength of a radio signal decreases over distance. The rate of decrease is called the path loss, and it is related to the wavelength of the signal. To calculate a simple path loss, use this equation: $$\text{Path loss} = \left(\frac{\lambda}{4\pi d}\right)^2$$ Ask students to complete Questions 1 and 2. Answers: 1. 5.17 x 10^{-12} meters or 5.17 picometers 2. Path loss is unitless (the meters from the wavelength and distance will cancel each other out)
Slide 7: Frequency  	When working with radio signals, engineers often describe a signal using its frequency instead of wavelength. For any electromagnetic wave (including radio waves), the frequency times wavelength equals the speed of light (300x10^6 meters/second). $$f\lambda = 300 \times 10^6 m/s$$ Ask students to complete Questions 3 and 4. Answers: 3. 1.5 x 10^9 hertz or 1.5 gigahertz 4. 5.7 x 10^{-8}
Slide 8: Gain and Power  	A key factor to know when working with a wireless device is the gain of its antenna. In simple terms, the gain tells you how well the antenna can transmit or receive signals at a frequency in a certain direction. The gain does not have a unit of measure. The power of the signal you receive (P_R) is a function of the power transmitted (P_T), the gain of the transmitter (G_T), the gain of your receiver (G_R) and the path loss. It is described by this equation:

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$$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi d)^2}$$

Ask students to complete Questions 5, 6 and 7.

Answers:

5. 7.124×10^{-6} watts or 7.124 microwatts
6. 1.24×10^{-8} watts or 12.4 nanowatts
7. 116,973.09 or 1.17×10^{5}

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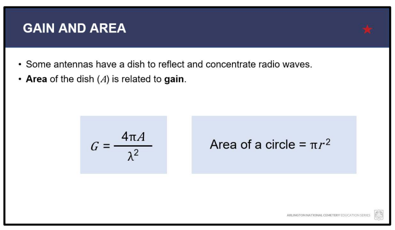
Slide 9: Gain and Area



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Some antennas have a dish to reflect and concentrate radio waves. The area of this dish is related to gain, as expressed by this equation:

$$G = \frac{4\pi A}{\lambda^2}$$

Ask students to complete Questions 8 and 9.

Answers:

8. 20,943.95 square meters
9. 81.65 meter radius

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EXTENSION ACTIVITIES:

- Find additional lesson plans and resources related to intelligence on the International Spy Museum website: https://www.spymuseum.org/education-programs/educators/educator-resources/
- Check out the science and math lessons involving satellites on the NASA Jet Propulsion Laboratory website: https://www.jpl.nasa.gov/edu/teach/tag/search/Satellites
- The National Cryptologic Museum offers in-person and virtual education events: https://www.nsa.gov/History/National-Cryptologic-Museum/Museum-Tours/

PLANNING TO VISIT ARLINGTON NATIONAL CEMETERY?

While at the cemetery, you can use the Military Intelligence walking tour to learn more about individuals who shaped the United States' modern intelligence programs.

SOURCES

Sellers, Jerry Jon, Wiley J. Larson, William J. Astore, Anita Shute, and Dale Gay. *Understanding Space: An Introduction to Astronautics: Third Edition.* New York: McGraw-Hill, 1994.

Sklar, Bernard. *Digital Communications: Fundamentals and Applications: Second Edition.* Upper Saddle River, New Jersey: Prentice Hall PTR, 2001.

INTELLIGENT INTELLIGENCE: SATELLITE IMAGERY

RESOLUTION FACTORS	
Aperture/lens diameter = D	$\text{Resolution} = \frac{2.44\lambda h}{D}$
Wavelength = λ	
Distance from object = h	

tera	giga	mega	kilo	hecto	deka		deci	centi	milli	micro	nano	pico
T	G	M	K	h	da		d	c	m	µ	n	p
10^{12}	10^{9}	10^{6}	10^{3}	10^{2}	10^{1}	10^{0}	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

1. You receive word that the enemy country of ABCDE has developed a new warplane that is 21 meters long. This new plane looks almost exactly like a type of passenger plane that is 18 meters long. If you wanted to look at images of airfields and pick out which planes are the warplanes and which are the passenger planes, what is the maximum resolution you could use to differentiate between the two?

   You would need a resolution of at least 3 meters.

2. You have multiple satellites you could use to take pictures of ABCDE's airfields, each with different lens sizes and distances from the earth. They all sense visible light with a wavelength of 532 nanometers. Which of these satellites could get you the resolution you need?

Satellite 1	Satellite 2	Satellite 3	Satellite 4
Distance: 750 kilometers	Distance: 200 kilometers	Distance: 500 kilometers	Distance: 600 kilometers
Diameter: 26 centimeters	Diameter: 2 centimeters	Diameter: 15.5 centimeters	Diameter: 28 centimeters

$$\text{Satellite 1 Resolution} = \frac{2.44\times(532\times10^{-9}\text{meters})\times750{,}000\text{ meters}}{.26\text{ meters}} = \textit{3. 74 meters, too large}$$

$$\text{Satellite 2 Resolution} = \frac{2.44\times(532\times10^{-9}\text{meters})\times200{,}000\text{ meters}}{.02\text{ meters}} = \textit{12.98 meters, too large}$$

$$\text{Satellite 3 Resolution} = \frac{2.44\times(532\times10^{-9}\text{meters})\times500{,}000\text{ meters}}{.155\text{ meters}} = \textit{4.19 meters, too large}$$

$$\text{Satellite 4 Resolution} = \frac{2.44\times(532\times10^{-9}\text{meters})\times600{,}000\text{ meters}}{.28\text{ meters}} =$$

2.78 meters; resolution small enough to differentiate between the two types of planes

3. Uh-oh! ABCDE has their own reconnaissance satellites, and you would like to know just what they are capable of seeing. You know that the satellites are at a distance of 700 kilometers from earth. Using additional intelligence, you can estimate their lens diameter to be half a meter. Assuming the ABCDE satellite is sensing visible light with a wavelength of 532 nanometers, what is its resolution?

$$\text{Resolution} = \frac{2.44 \times (532 \times 10^{-9} \text{meters}) \times 700{,}000 \text{ meters}}{.5 \text{ meters}} = \textit{1.82 meters or 182 centimeters}$$

4. What would be the same satellite's resolution if it was sensing short-wave infrared, with a wavelength of 1 micrometer (also called a micron)?

$$\text{Resolution} = \frac{2.44 \times (1 \times 10^{-6} \text{meters}) \times 700{,}000 \text{ meters}}{.5 \text{ meters}} = \textit{3.42 meters or 342 centimeters}$$

5. A new reconnaissance satellite is being developed that has a resolution of 2 millimeters. You are going to be able to count the number of ants on an anthill! The satellite will sense a wavelength of 5 microns, and be at a distance of 330 kilometers. What will the aperture diameter be?

$$.002 \text{ meters} = \frac{2.44 \times (5 \times 10^{-6} \text{meters}) \times 330{,}000 \text{ meters}}{\text{D}}$$

D = 2,013 meters or 2.013 kilometers

INTELLIGENT INTELLIGENCE: RADIO COMMUNICATIONS

tera	giga	mega	kilo	hecto	deka		deci	centi	milli	micro	nano	pico
T	G	M	K	h	da		d	c	m	µ	n	p
10^{12}	10^{9}	10^{6}	10^{3}	10^{2}	10^{1}	10^{0}	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

FORMULA ELEMENTS	UNIT OF MEASURE
Wavelength = λ	Meters (m)
Distance = d	Meters (m)
Frequency = f	Hertz (hz)
Gain = G	
Power = P	Watts (w)
Area = A	Square meters (m^2)
Radius = r	Meters (m)

FORMULAS
$\text{Path loss} = \left(\frac{\lambda}{4\pi d}\right)^2$
$f\lambda = 300 \times 10^6 m/s$
$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi d)^2}$
$G = \frac{4\pi A}{\lambda^2}$
$\text{Area of a circle} = \pi r^2$

1. Calculate the path loss given a wavelength of 2 meters and distance of 70 kilometers.

$$\text{Path loss} = \left(\frac{2 \text{ meters}}{4\pi * 70{,}000 \text{ meters}}\right)^2$$ = ***5.17 x 10^{-12} meters or 5.17 picometers***

2. What are the units of path loss?

Path loss is unitless

3. What is the frequency of a signal with a wavelength of 20 centimeters?

$$\text{Frequency} = \frac{300 \times 10^6}{.2}$$

Frequency = 1.5 x 10^9 hertz or 1.5 gigahertz

4. What is the path loss of a signal at 100 megahertz over 1 kilometer?

$$\text{Wavelength} = \frac{300 \times 10^6}{100 \times 10^6} = 3 \text{ meters}$$

$$\text{Path loss} = \left(\frac{3 \text{ meters}}{4\pi * 1{,}000 \text{ meters}}\right)^2$$ = ***5.7 x 10^{-8}***

5. You are in the field transmitting with a power of 10 watts, gain of 10, and wavelength of 3 meters. The receiver is 6 kilometers away and has a gain of 45. What is the power received?

$$P_R = \frac{10 \text{ watts} \times 10 \times 45 \times 3^2 \text{ meters}}{(4\pi \times 6{,}000 \text{ meters})^2}$$ = ***7.12 x 10^{-6} watts or 7.12 microwatts***

6. What if you used the same transmitter and receiver, but transmitted at a frequency of 2.4 gigahertz? What would be the power received?

$$\text{Wavelength} = \frac{300 \times 10^6}{2.4 \times 10^9} = .125 \text{ meters}$$

$$P_R = \frac{10 \text{ watts} \times 10 \times 45 \times .125^2 \text{ meters}}{(4\pi \times 6{,}000 \text{ meters})^2}$$ = ***12.4 x 10^{-8} watts or 12.4 nanowatts***

7. You want to intercept the communications of terrorists using handheld radios at a frequency of 200 megahertz, with a transmit power of 10 watts and gain of 1.5. You are 50 kilometers away, and your receiver requires 10 microwatts of power. In order to receive the terrorists' signals, what gain would your antenna need to have?

$$\text{Wavelength} = \frac{300 \times 10^6}{200 \times 10^6} = 1.5 \text{ meters}$$

$$10 \times 10^{-6} = \frac{10 \text{ watts} \times 1.5 \times G_R \times 1.5^2 \text{ meters}}{(4\pi \times 50{,}000 \text{ meters})^2}$$

G_R = 116,973.09

8. Using the gain of the receiver in Question 7, what would be the area of your dish?

$$116{,}973.09 = \frac{4\pi A}{1.5^2}$$

$$116{,}973.09 \times 2.25 = 4\pi A$$

Area = 20,943.95 square meters

9. Assuming the dish is a circle, what would be the radius of the dish?

$$20{,}943.95 \text{ square meters} = \pi r^2$$

$$\frac{20{,}943.95}{\pi} = r^2$$

$$r = \sqrt{(20{,}943.95/\pi)}$$

Radius = 81.65 meters